Hard Optimization and Related Rates Problems

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1 Optimization

Problem 1

Find the equation of the line through $(2, 4)$ that cuts off the least area from the first quadrant.

Problem 2

Suppose Peyam's utility function is given by:

$$U(\theta) = \frac{\mu\, C}{\mu \sin(\theta) + \cos(\theta)}$$

where C is a universal cake-constant and μ is the intensity of happiness.

Note: Think of θ here as the time of the year. So $\theta = 0$ corresponds to the beginning of January 1st, and $\theta = 2\pi$ corresponds to the end of December 31st.

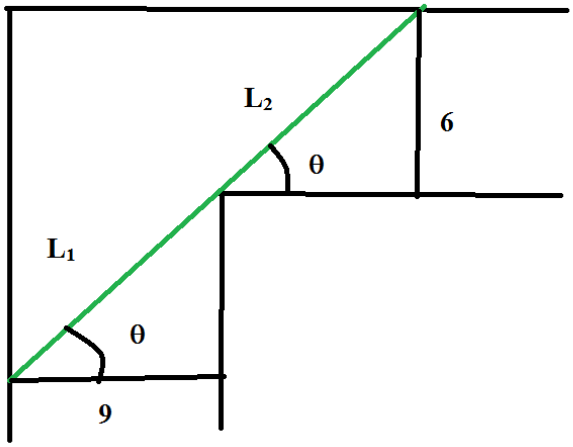
Show that Peyam is happiest precisely when $\tan(\theta) = \mu$.

Problem 3

A steel pipe is being carried down a hallway 9 feet wide. At the end of the hall there is a right-angled turn into a narrower hallway 6 feet wide. What is the length of the longest pipe that can be carried horizontally around the corner?

Note: See picture on the next page

1A/Math 1A - Fall 2013/Homeworks/Pipe.png



2 Related Rates

Problem 4

Suppose that the minute hand of a clock is 15 mm long and the hour hand is 12 mm. How fast is the distance between the hour hand and the minute hand changing at 2 pm?

Note: This time, assume that both the hour and minute hands are moving!

Problem 5

A water tank has the shape of a horizontal cylinder with radius 1 and length 2. If water is being pumped into the tank at a rate of $\frac{1}{6}$ m^3 per minute, find the rate at which the water level is rising when the water is $\frac{1}{2}$ m deep.